ESSENTIAL EQUATIONS OF TURBOMACHINES

author ŠKORPÍK, Jiří (LinkedIn.com/in/jiri-skorpik)

issue date September 2009; September 2022 (2nd ed.)

title Essential equations of turbomachines

proceedings transformacni-technologie.cz/index_en.html#**turbomachinery**

provenance Brno (Czech Republic)

email skorpik.jiri@email.cz

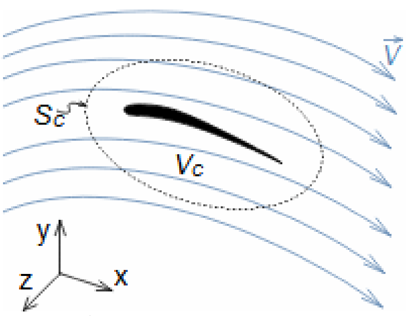
EQUATION FOR CALCULATING FORCES ACTING ON MACHINE SURFACES FROM FLUID FLOW

The forces acting from a fluid flow on a machine surfaces can be determined from the MOMENTUM CHANGE THEOREM. Its special form is also used to calculate the FORCES ACTING ON BLADES in a blade cascade from a fluid flow, a classical problem in turbomachinery.

~
Forces acting on fluid inside investigated (control) volume

Forces acting from a fluid flow on machine surfaces can be determined from the momentum change theorem (Newton's second law). According to the momentum change theorem, a change in fluid momentum over time is equal to the sum of the external forces acting on the fluid in the control volume. In the case of applying this law to a fluid enclosed in a control volume V_C (FIGURE 255), the external forces considered are: the pressure forces from the surrounding fluid at the boundaries of the control volume F_p, the weight of the fluid in the control volume F_h, and the forces exerted by the bodies inside and at the boundaries of the control volume F_b. The change in momentum of the fluid inside the control volume is also equal to the difference of the product of velocity and mass flow between the inlet and outlet of the control volume.

255:
Theorem of momentum change



$$\frac{d\bar{M}}{dt}=\bar{F}_b+\bar{F}_h+\bar{F}_p=\int_{S_c}\bar{V}\,d\dot{m}$$

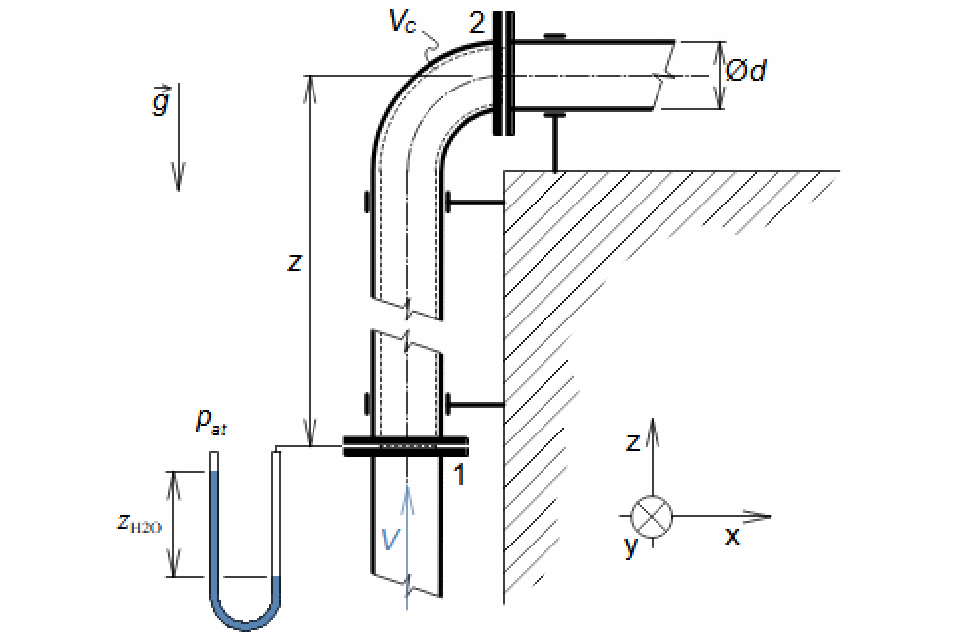
$$\bar{F}_h=\int_{V_c}(\bar{g}+\bar{a}_r+\bar{a}_c)\,dm$$

$$\bar{F}_p=\int_{S_c}p\,d\bar{S}_c$$

V_C [m^3] control volume; S_C [m^2] surface of control volume boundary; V [m·s^{-1}] velocity of working fluid; M [N·s] momentum of fluid inside control volume; t [s] time; F_b [N] resultant of forces acting on working fluid from bodies inside and on control volume boundary; F_h [N] weight of working fluid inside control volume; F_p [N] forces from pressure of surrounding fluid on surface of control volume; $\dot{m}$ [kg·s^{-1}] mass flow; g [m·s^{-2}] gravitational acceleration; a_r [m·s^{-2}] centrifugal acceleration; a_C [m·s^{-2}] Coriolis acceleration; p [Pa] pressure; m [kg] mass. The derivation of these equations assuming steady fluid flow through the control volume is shown in APPENDIX 255.

Problem 254:

What force acts on the pipe between flanges due to fluid flow (see figure)? The inner pipe diameter is 23 mm, the flange height difference is 1,2 m, static pressure in the pipe versus outside pressure is 2 m water column, flow velocity is 4 m·s^{-1}, and it's frictionless flow. Water is flowing in the pipe. You are considering frictionless flow. The solution of this problem is shown in Appendix 254.



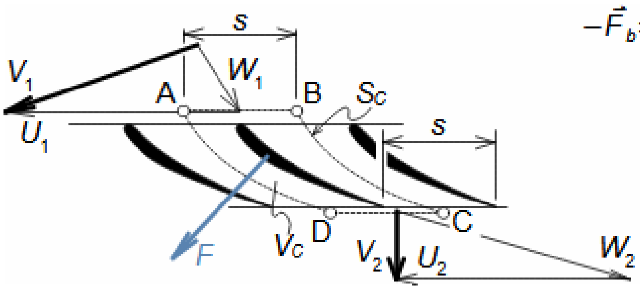
d [m] pipe diameter; p_{at} [Pa] atmospheric pressure; *z* [m] altitude coordinate.

Force on blade from fluid flow

When calculating turbomachine blade forces, define control volume boundaries where parameters for momentum change theorem are known. Therefore, the control volume in Figure 196 is defined to pass through the center of the blade passage, or the boundaries AD and BC are spaced apart by the pitch of blades. The boundaries AD and BC are the expected streamlines of the relative velocities *W* of the velocity triangle of this blade cascade. The blade passages are equal in a single blade cascade, so that the action of the pressure forces at the AD boundaries cancel with the action of the pressure forces at the BC boundaries. The integration of the product of the absolute velocity *V* and the mass flow is also cancelled at these boundaries, see Equation 196.

196:

$$-\vec{F}_b=\vec{F}=\vec{V}_1\cdot\dot{m}-\vec{V}_2\cdot\dot{m}+\vec{F}_h+\vec{F}_p$$



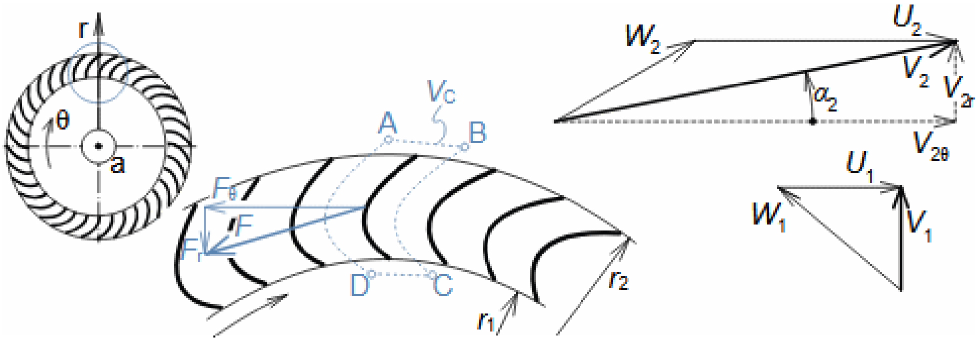
Both velocities and forces are vector quantities, but the arrow above the velocity symbols in the velocity triangle is usually not shown. F [N] resultant of forces acting on blade; W [m·s^{-1}] relative velocity; U [m·s^{-1}] blade speed; $m^{\bullet}$ [kg·s^{-1}] amount of working fluid flowing through control volume; s [m] pitch of blade. The derivation of this equation for the assumption of stationary flow through the control volume is shown in APPENDIX 196.

Spatial components of force

Typical for the investigation of forces in a turbomachine stage is the use of cylindrical coordinate system. The force F in the cylindrical coordinate system has three spatial components, namely a component in the radial direction F_r, in the tangential direction F_θ (this force produces torque) and in the axial direction F_a (it causes stress on the rotor in the axial direction and is collected by a thrust bearing) - these force components are abbreviated as radial, tangential and axial forces.

Problem 584:

Determine the force and its components from the fluid flow acting on the radial fan blade. 88,8 kg·h^{-1} of air flows through the fan, the pressure p_1 upstream of the impeller is atmospheric, the pressure difference between the impeller inlet and outlet is insignificant and the number of blades is 52. The other parameters are: r_1=32,5 mm, r_2=37,5 mm, V_1=3,4 m·s^{-1}, V_2=9,34 m·s^{-1}, α_2=18,4°. The width of the impeller is 30 mm. The solution of this problem is shown in APPENDIX 584.



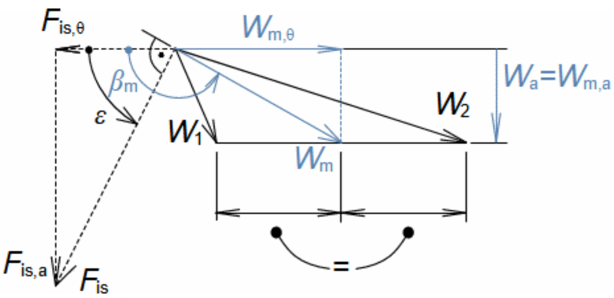
r [m] r [m] radius; α [°] angle of absolute velocity.

Mean aerodynamic velocity

The force acting on the blade is approximately perpendicular to the mean aerodynamic velocity in the blade cascade W_m, which is the mean of the relative velocities at the inlet W_1 and outlet W_2. Respectively, it can be shown (see EQUATION 248, p. 5) that the resulting force on the blade from the incompressible fluid flow F is perpendicular to the mean aerodynamic velocity W_m in lossless flow.

248:

Definition of mean aerodynamic velocity in blade cascade and its relation to force vector acting on elementary blade (blade length dr)



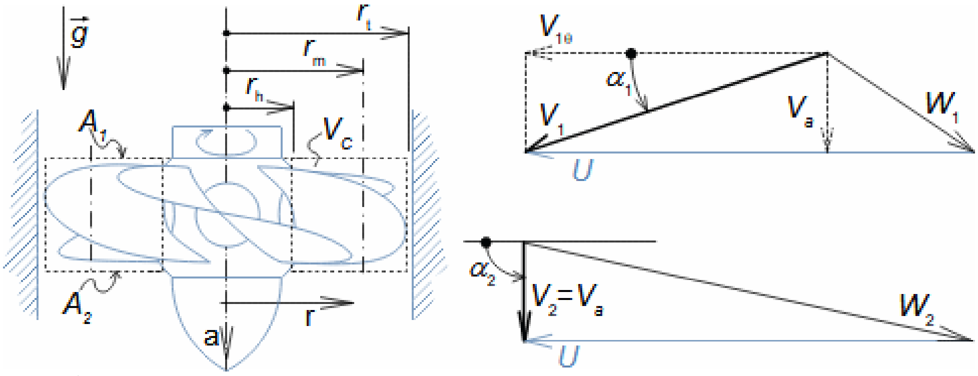
$$\vec{W}_m=\frac{1}{2}(\vec{W}_1+\vec{W}_2)$$

$$\tan\varepsilon=\frac{1}{-\tan\beta_m}\Rightarrow F_{is}\perp W_m$$

W_m [m·s^{-1}] mean aerodynamic velocity in blade cascade; β_m [°] angle of mean aerodynamic velocity; ε [°] angle of resultant force. This equation is derived for the elementary blade length Δr and the axial blade cascade in Appendix 248 and its validity is limited to incompressible flow without losses (isoentropic - index $_{is}$).

Problem 583:

Calculate force on a Kaplan turbine rotor blades from water flow and outlet pressure p_2. Blade tips radius: 1850 mm, hub radius: 985 mm, tangential velocity: 15,3 m·s^{-1}, axial velocity: 13 m·s^{-1}, turbine rotational speed: 230,8 min^{-1}. 56 m water column above turbine. Velocity triangle shapes at mean radius shown in attached figure. The solution of this problem is shown in Appendix 583.



A [m^2] flow area. The index $_h$ indicates hub of the blade, the index $_m$ indicates the mean square radius of the blade, the index $_t$ indicates the tip of the blade.

EQUATIONS FOR CALCULATING ENERGY DISTRIBUTION IN TURBOMACHINE STAGE

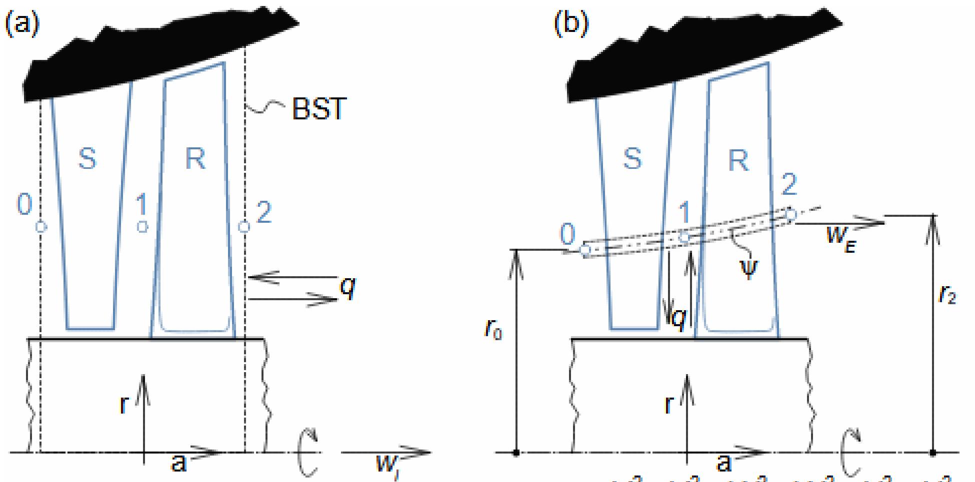
The proposal of the energy distribution or transformation in the turbomachine stage is based on two directions. In the direction perpendicular to the meridional direction, an Euler work distribution, which is the local value of the internal work, is proposed. In the meridional direction, the proposal of the reaction, which describes the distribution of energy transformations between the stator and the rotor of the stage, decides the characteristics of the stage.

Euler work

The Euler work is the fluid work transferred to the blades in the surroundings of the streamline under investigation, see FIGURE 318b. The difference with the internal work of the stage w_i is that the internal work of the stage is the mean work of all the working fluid flowing through the stage (including gaps) and can be determined from the complete energy balance of the stage, see FIGURE 318a. So some of the fluid will do more Euler work than others, but their mean is w_i. For actual stages, the largest Euler work is in the core of the flow (at the mean diameter of the blades), where the losses are smallest. Conversely, at the edges of the blades, or near their roots and tips, the Euler work is smallest due to high frictional losses and internal leakage. The Euler work can be determined from the velocity triangles on the streamline under investigation, see EQUATION 318c - Euler turbomachinery equation.

318:

Difference between Euler work and internal work of stage



$$\text{(c)}\quad w_E=U_1\cdot V_{1\theta}-U_2\cdot V_{2\theta}=\omega(r_1\cdot V_{1\theta}-r_2\cdot V_{2\theta})=\frac{V_1^2-V_2^2}{2}+\frac{W_2^2-W_1^2}{2}+\frac{U_1^2-U_2^2}{2}$$

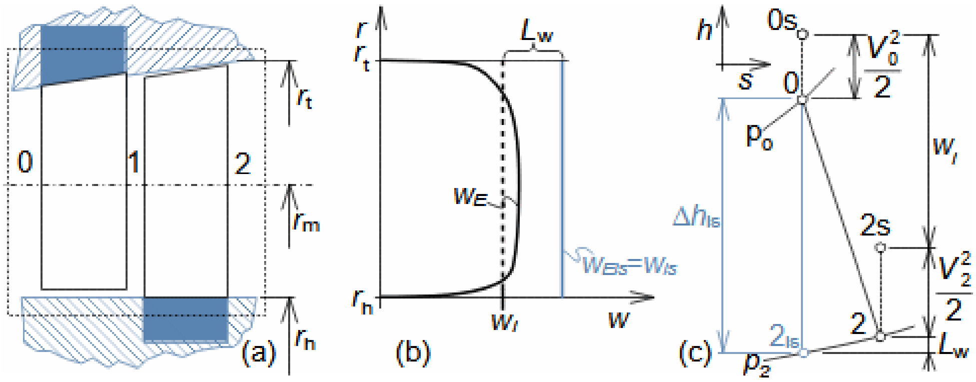
w_i [J·kg^{-1}] internal work of stage; w_E [J·kg^{-1}] Euler work in surroundings of investigated streamline; q [J·kg^{-1}] shared heat with surroundings; ω [°] angular speed. BST-stage boundary; S-stator blade cascade; R-rotor blade cascade, ψ-streamline. The derivation of the Euler turbomachinery equation for the assumption of stationary flow is shown in APPENDIX 318.

Euler efficiency

Similarly, the stage efficiency can be determined in two ways, either to the idela/real Euler work (Euler efficiency) or to the idela/real internal work (internal stage efficiency), as done in PROBLEM 923 (p. 8).

Problem 923:

Calculate the Euler work and Euler efficiency at the mean radius of the axial stage of the steam turbine and the internal work and efficiency of this stage. The stage has been designed by 1D calculation hence it has straight blades. The meridional velocity is constant ($V_{0a}=V_{2a}$). The isentropic gradient of the stage is 21,3 kJ·kg^{-1}. The calculated total loss of the stage is 6 kJ·kg^{-1}. The parameters of the velocity triangles at the mean radius are: $V_1=W_2=148{,}68$ m·s^{-1}, $V_0=V_2=W_1=63{,}249$ m·s^{-1}, $U_1=U_2=102{,}1$ m·s^{-1}. The solution of this problem is shown in APPENDIX 923.

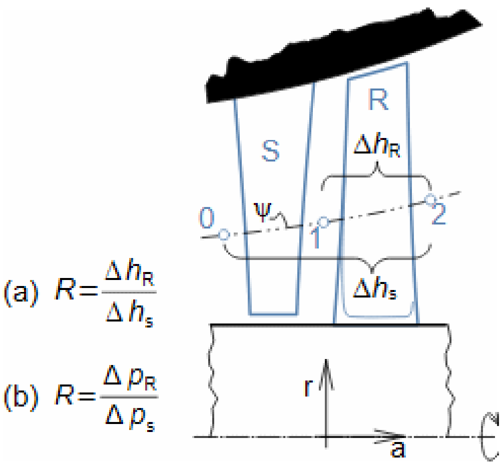


(a) cross-section of stage; (b) Euler work along length of blades; (c) energy balance of stage in *h*-*s* chart. *h* [J·kg^{-1}] enthalpy; *s* [J·kg^{-1}·K^{-1}] entropy; w_{is} [J·kg^{-1}] internal work of the stage at isentropic expansion (expansion without losses); $w_{E,is}$ [J·kg^{-1}] Euler work at flow without losses; L_w [J·kg^{-1}] internal losses of the stage. The index $_s$ denotes the total state. A reaction is determined to a specific streamline (radius) similar to Euler work.

~
Reaction

A reaction is the ratio between the change in the static enthalpy in the rotor blade cascade and the change in the stagnation enthalpy of the stage (FORMULA 344), or the change in the static enthalpy of the stage - depending on convention. Thus, it describes the distribution of the energy transformation between the stator and rotor blade cascades of the stage.

344:
Definition of reaction

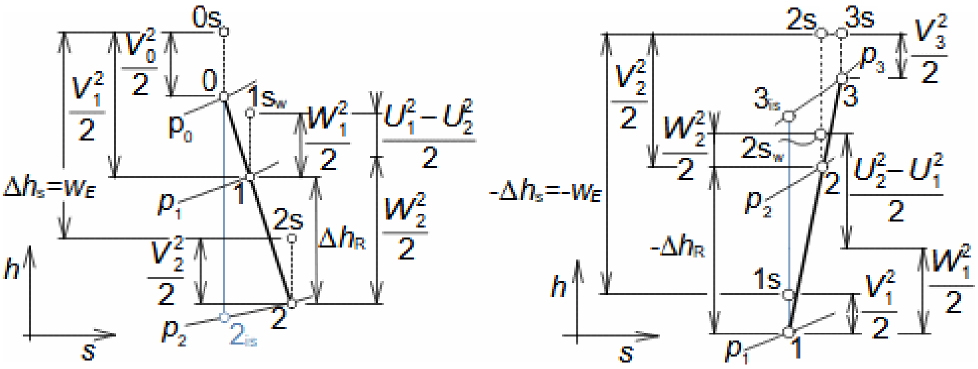


(a) general definition of reaction; (b) special definition of reaction used for hydraulic machines. Δh_s [J·kg^{-1}] difference between stagnation enthalpy at inlet to stage and outlet from stage; Δh_R [J·kg^{-1}] difference between static enthalpy at inlet to rotor blade cascade and outlet from rotor blade cascade; Δp_s [Pa] difference between stagnation pressure at inlet to stage and outlet from stage; Δp_R [Pa] difference between pressure at inlet to rotor blade cascade and outlet from rotor blade cascade.

h-s charts of turbomachine stages

To calculate the reaction, it is important to know the construction of the *h-s* chart, from which the differences in specific enthalpies Δh_s and Δh_R can be determined (see PROBLEM 179, PROBLEM 1030). *h-s* charts and a description of their construction are showen in FIGURE 713. In the case of hydraulic machines, the required pressure differences Δp_s and Δp_R can also be determined from Bernoulli equation for relative flow, see PROBLEM 256 (p. 8) and PROBLEM 1030.

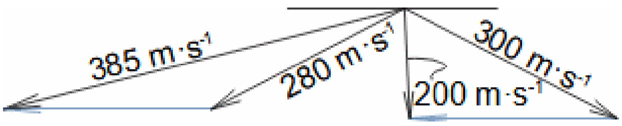
713:



left-turbine stages; right-working machine stages. These *h-s* charts are constructed under the assumption of adiabatic flow without gravity effect. $1s_w$, $2s_w$ denote the overall condition with respect to the relative motion at the rotor inlet and outlet. A detailed description of the construction of the *h-s* charts is shown in APPENDIX 713.

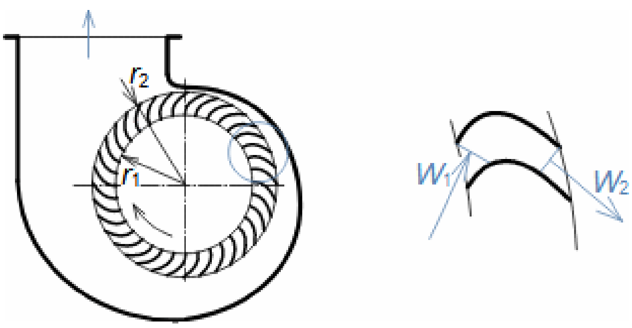
Problem 179:

Calculate the reaction of the axial stage of a steam turbine. If you know the velocity triangle. The solution of this problem is shown in APPENDIX 179.



Problem 1030:

Determine the reaction of a radial fan with forward curved blades if the stagnation pressure increase in the fan is 135 Pa, the working gas density is 1,2 kg·m^{-3}, the blade speed at the outlet of the rotor is 10 m·s^{-1} and the blade speed at the inlet of the rotor is 8,7 m·s^{-1}. The rotor blade passages are designed for equality of relative velocities (W_1=W_2). The solution of this problem is shown in APPENDIX 1030.



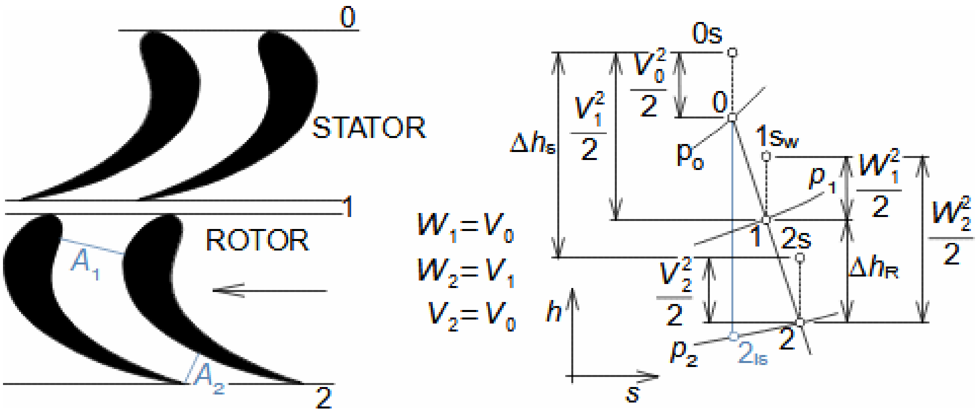
Distribution of reaction

Axial stages have variable reaction along the length of the blades, see PROBLEM 736 (p. 12), but radial stages can be designed so that the reaction is the same for any streamline – this is used in particular to optimize internal losses in Francis turbines and radial stages of pumps, see the problem in the article WATER TURBINES.

Reaction on mean radius

Most stages are designed so that the reaction at the mean radius of the blades is equal to 0,5 (even a little higher for radial stages due to the difference in blade speeds on the rotor), since at maximum Euler work the absolute velocities in the stator passages are about the same as the relative velocities in the rotor passages, and hence the distribution of losses between stator and rotor is uniform, see FIGURE 280.

280:
Example of turbine axial stage blade passages with reaction of 0,5



With the same enthalpy differences between the stator and rotor, the velocity triangles are symmetrical and the shape of the blade passages ($A_1 \neq A_2$) is also symmetrical. $1s_w$ is denoted relative stagnation state of working fluid at rotor inlet.

Influence of reaction to pressure

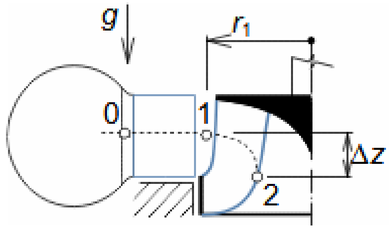
Laval turbines and Pelton turbines show minimal the reaction. In for case working machines, low reactions are used in certain fans also. At low reaction, compressive force on the rotor is small, calling these stages pressureless or impulse stages. Conversely, stages with significant reaction yield greater compressive force, termed overpressure or reaction stages.

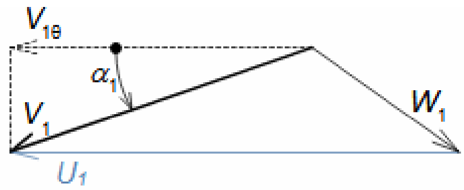
Influence of reaction on camber

In axial stages, as reaction increases, blade camber decreases (required momentum change drops), reducing sensitivity to flow separation.

Problem 256:

Calculate the reaction of the Francis turbine at its mean radius. The radius of the impeller at the inlet is 1 m. The absolute velocity in front of the impeller is 35 m·s^{-1}, behind the impeller is 12 m·s^{-1} (it has no tangential component). The turbine rotational speed is 375 min^{-1}. The angle of absolute velocity is 20°. The height difference between the inlet and outlet of the impeller is 0,8 m. The density of water is 1000 kg·m^{-3}. The solution of this problem is shown in APPENDIX 256.





EQUATIONS FOR CALCULATING VELOCITY DISTRIBUTION AT IDEAL FLOW IN TURBOMACHINE

Ideal flow equations are derived for ideal fluid flow without internal losses. he analytical model of frictionless flow is most often the theory of potential flow, and in the case of flow around an axis, its special case of AXISYMMETRIC POTENTIAL FLOW. These equations are supplemented by energy balance equations and EULER EQUATION OF HYDRODYNAMICS. Although they are ideal flow equations, they are crucial for basic design of turbomachinery flow components, prediction of properties, analysis of the effect of flow component shape on internal losses of the machine, and understanding the causes of failures or problem operation of turbomachines.

~
Axisymmetric potential (non-vortex) flow conditions

For potential flow, the curl velocity vector must be zero throughout the volume (EQUATION 586a). For axisymmetric flow, gradients of velocity components in the tangential direction (EQUATION 586b) must be zero in a cylindrical coordinate system, because the tangential coordinates are closed and the velocity at the origin of the tangential axis must be identical to that at the end of the coordinates. These conditions yield special velocity EQUATIONS 586c-h, applicable to other fluid quantities.

586:
Mathematical notation of conditions for axisymmetric potential flow

$$\left.\begin{array}{l}\text{(a)}\ \mathrm{rot}\ \vec{V}=0\\[2ex] \text{(b)}\ \dfrac{1}{r}\dfrac{\partial V_{r,\theta,a}}{\partial\theta}=0\end{array}\right\}\Rightarrow \underset{\text{(c)}}{\frac{\partial V_r}{\partial\theta}=0},\ \underset{\text{(d)}}{\frac{\partial V_\theta}{\partial\theta}=0},\ \underset{\text{(e)}}{\frac{\partial V_a}{\partial\theta}=0},\ \underset{\text{(f)}}{\frac{\partial V_\theta}{\partial a}=0},\ \underset{\text{(g)}}{\frac{\partial V_r}{\partial a}=\frac{\partial V_a}{\partial r}},\ \underset{\text{(h)}}{\frac{1}{r}\frac{\partial(r\cdot V_\theta)}{\partial r}=0\Rightarrow r\cdot V_\theta=C}$$

θ [°] azimuth in cylindrical coordinate system; C [$m^2 \cdot s^{-1}$] constant (e.g., proposed magnitude of product of tangential component of absolute velocity V_θ on mean radius). The modification of these equations is shown in APPENDIX 586.

Distribution of Euler work

The product $r \cdot V_\theta$ is called the circulation of the tangential component of the velocity, which is constitutive, so it has the same properties as the irrotational vortex [Škorpík, 2023, p. 1.40]. If the circulation is constant, then the difference of the circulations in front of and behind the rotor is also constant and then also according to the Euler work equation (Equation 318, p. 7) the Euler work of the potential flow will be constant along the length of the blades, see Problem 736.

Spiral paths

The equations of axisymmetric potential flow can also be applied to spiral paths, for example in spiral casings (Problem 282, p. 13) or in bladeless stators (Problem 407, p. 14).

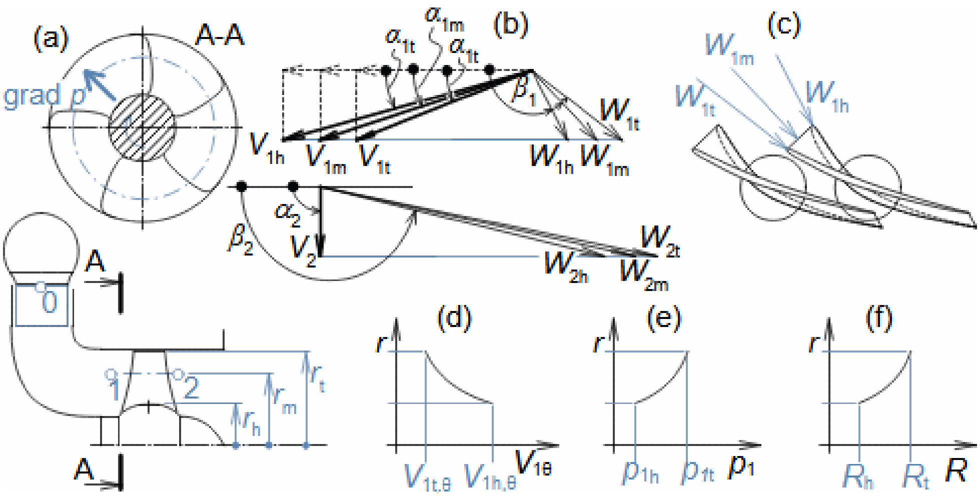
~

Euler equation of hydrodynamics

The Euler equation of hydrodynamics for potential flow can also be used to calculate other state variables, for example, the article Internal fluid friction and boundary layer development [Škorpík, 2023b]. From this equation one can read, among other things, that the pressure gradient of a potential flow without the effect of gravitational acceleration cannot have a tangential component, because the gradient of velocity respectively kinetic energy does not have one either, see Problem 736.

Problem 736:

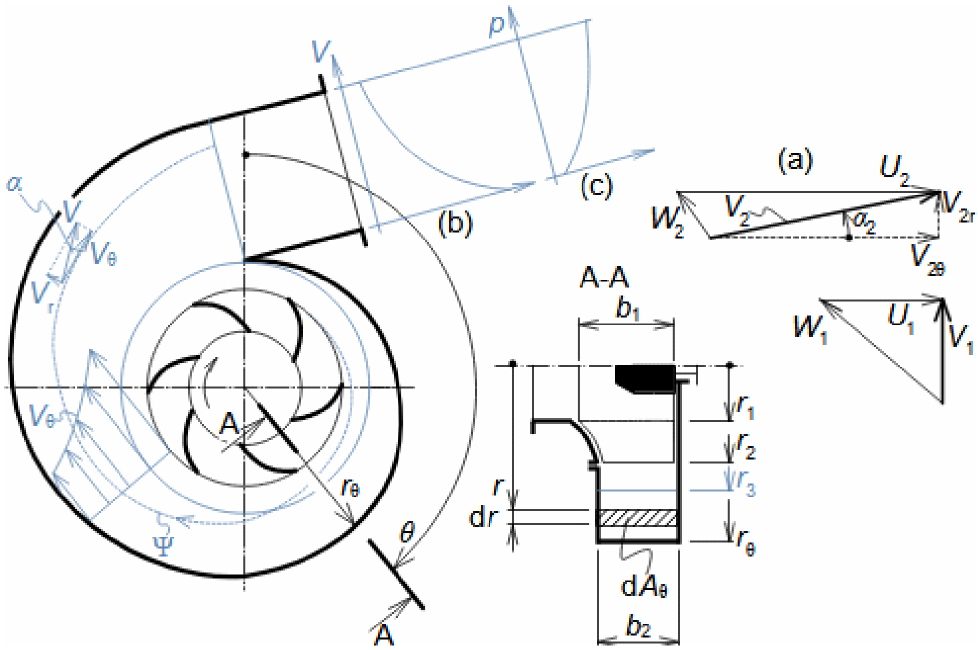
Calculate the parameters of the velocity triangle, pressure and reaction at the hub, mean square radius and tip of the Kaplan turbine blade. The required Euler work is 548 J·kg^{-1}. The rotor dimensions, rotational speed, axial velocity at the mean square radius, and pressure behind the rotor are the same as in Problem 583. The absolute velocity at the rotor exit has only the axial direction. Also determine the pressure gradient in front of and behind the turbine rotor. Consider the potential flow of an ideal fluid. The solution of this problem is shown in Appendix 736.



(a) pressure gradient in front of rotor; (b) changes in absolute and relative velocities at investigated radii; (c) effect of change in relative velocities on shape of blade passage, or blade twist; (d) distribution of tangential component of absolute velocity in front of rotor; (e) distribution of pressure in front of rotor; (f) distribution of reaction along blade length. β [°] angle of relative velocity.

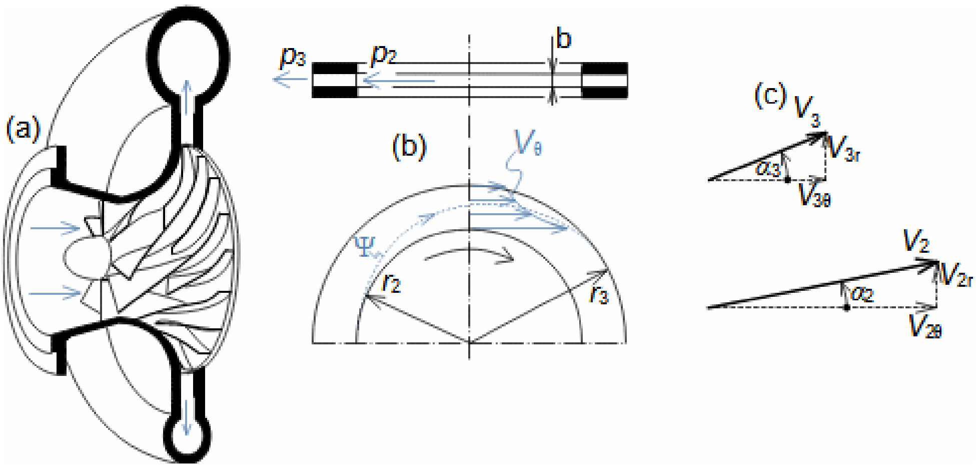
Problem 282:

The purpose of the spiral casings of radial machines is to discharge or feed the working fluid from/to the blade section. The flow in such a casing has a spiral path. The figure shows a section of a radial fan with backward curved blades and a spiral casing - suggest the dimensions of this spiral casing if there is a bladeless diffuser between it and the rotor. Determine the pressure at the outlet of the bladeless diffuser (between radii r_2 and r_3). Prove that when an incompressible fluid flows through a radial duct of constant width b, the spiral path is a logarithmic spiral. Discuss the effect of internal friction in the fluid on the shape of the spiral path. What is the velocity and pressure distribution at the exit of the spiral casing? Consider incompressible potential flow. Discuss the effect of the casing width on the radius of the casing. The parameters of the fan are R=0,65; r_3=215 mm; r_2=170 mm; r_1=118,5 mm; b_2=101,5 mm; b_1=120 mm; N=1360 min^{-1}. The increase in stagnation pressure in the fan is 500 Pa. The air flow through the fan is 1200 m^3·h^{-1} and its stagnation suction pressure is atmospheric at a density of 1,2 kg·m^{-3}. The solution of the problem and other conclusions are shown in Appendix 282.



(a) velocity triangles; (b) velocity profile at outlet of spiral casing; (c) pressure profile at outlet of spiral casing. Ψ-spiral absolute velocity trajectory.

Problem 407: Design the outlet radius of the bladeless stator of a radial compressor, which is drawn in the figure. The stage parameters are: $V_{2\theta}$=300 m·s^{-1}, V_{2r}=90 m·s^{-1}, r_2=33 mm, p_2=200 kPa, t_2=82,9 °C. The stator pressure rise is 80 kPa. The working gas is air. Find also whether the angle between the absolute velocity in the bladeless stator and its tangential component (between radii r_2 and r_3) changes. Consider the compressible potential flow. The solution of this problem is shown in Appendix 407.



(a) rotor-bladeless stator assembly and inlet and outlet casing; (b) bladeless stator section; (c) absolute velocity at the inlet and outlet of the bladeless stator.

REFERENCES

ŠKORPÍK, Jiří, 2024, Technická termomechanika, *transformacni-technologie.cz*, Brno, transformacni-technologie.cz/technicka-termomechanika.html.

ŠKORPÍK, Jiří, 2023, Technická matematika, *transformacni-technologie.cz*, Brno, transformacni-technologie.cz/technicka-matematika.html.

ŠKORPÍK, Jiří, 2023b, Internal fluid friction and boundary layer development, *transformacni-technologie.cz*, Brno, transformacni-technologie.cz/internal-fluid-friction-and-boundary-layer-development.html.

ŠKORPÍK, Jiří, 2023c, Flow of gases and steam through nozzles, *transformacni-technologie.cz*, Brno, transformacni-technologie.cz/flow-of-gases-and-steam-through-nozzles.html.

ŠKORPÍK, Jiří, 2023d, Flow of gases and steam through diffusers, *transformacni-technologie.cz*, Brno, transformacni-technologie.cz/flow-of-gases-and-steam-through-diffusers.html.