14. RELATION BETWEEN SPECIFIC SHAFT WORK AND INTERNAL WORK OF TURBOMACHINE STAGE

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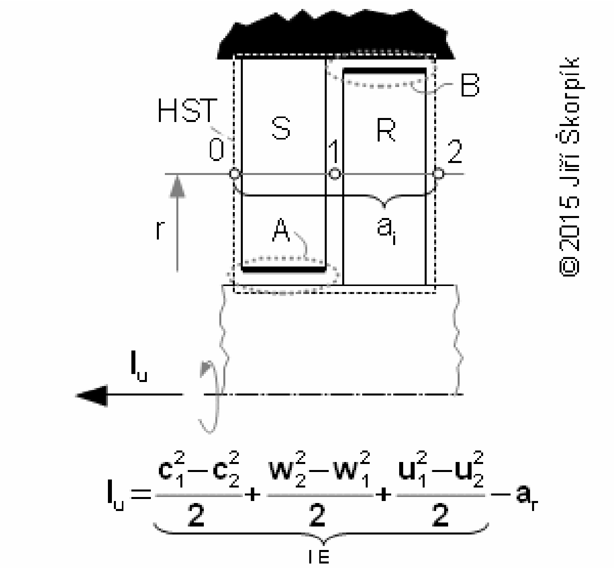
Introduction

This article follows the article 12. Essential equations of turbomachines and 13. Energy balances of turbomachines. In these articles are defined quantities a specific shaft work $\mathbf{l_u}$ and a specific internal work of turbomachine stage $\mathbf{a_i}$.

The specific internal work of stage is corresponded work of the working fluid inside stage and it is calculated from difference of stagnation states between the inlet and the exit of stage. The specific work shaft is corresponded work of the working fluid transformed to the torque of the shaft and it is calculated from a velocity triangels in front of and behind the rotor and from a friction between the rotor and the working fluid so called rotor friction losses:

Rotor friction losses

The specific shaft work of real machines is influenced by friction between the rotor and the working fluid so called rotor friction losses:



1.318 *Difference between work shaft and internal work of stage.*
HST volume of stage; **r** [m] radius; $\mathbf{l_u}$ [J·kg^{-1}] specific work shaft of stage on radius $r^{(1)}$; **c** [m·s^{-1}] absolute velocity; **w** [m·s^{-1}] relative velocity; **u** [m·s^{-1}] circumference velocity; $\mathbf{a_r}$ [J·kg^{-1}] rotor friction losses of stage$^{(2)}$; $\mathbf{l_E}$ [J·kg^{-1}] specific work of working fluid during flow through rotor blade row without rotor friction losses; $\mathbf{a_i}$ [J·kg^{-1}] specific internal work of stage; **A**, **B** areas of development friction losses under friction between rotor and working fuid. **S** stator blade row; **R** rotor blade row.

$^{(1)}$*Remark*
The specific shaft work can be changed along the lenght of blade. Also the pressure is changed along the lenght of blade under pressure gradient.

$^{(2)}$*Rotor friction losses*
The friction is function of a construction of the stage, it is the biggest in stages with a disc rotor (e.g. the one-stage Laval turbine or radial stages), this friction is usually negligible for stages with drum rotors.

The rotor friction losses of the stage is a consumed work, which is transformed on heat. This friction heat heatings the working fluid in surrounding:

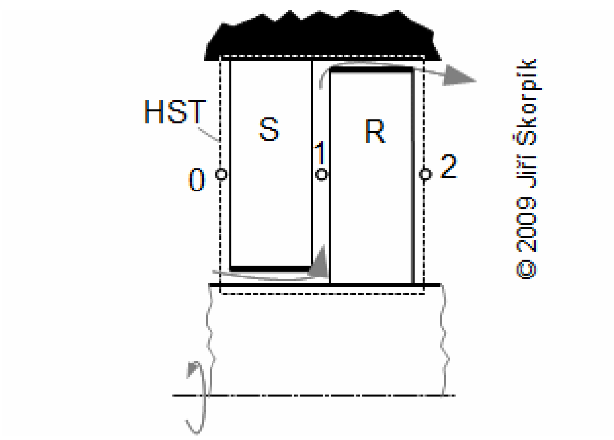
$$a_r = \delta \cdot a_r + (1-\delta) a_r$$

2.934 *Subdividing of heat flow from rotor friction.*

δ [-] coefficient subdividing of heat flow from rotor friction; $\boldsymbol{\delta \cdot a_r}$ [J·kg^{-1}] heat flow to part of machine (heat extraction to surrounding); $\boldsymbol{(1-\delta)a_r}$ [J·kg^{-1}] heat flow to working fluid.

From equation of specific internal work of turbomachine is evident heat $\delta \cdot a_r$ increases reajected heat to surroundings and heat $(1-\delta)a_r$ increases internal heat at the exit of stage u_e respective enthalpy i_e. This decreases the specific internal work and the specific shaft work same, therefore for now be can $l_u = a_i$.

The conclusion of the previous paragraph was formulated on the assumption that the working fluid flow only through blade passage at development only the profile losses and rotor friction losses without the other losses of the stage. But the turbomachine stage is a classically engineering product, which usually is not perfectly, therefore there are other losses[3] inside stages also e.g. a portion of the working fluid can flow outside the blade passages (leaks, construction gap) etc.:



3.1089 *Example of flow through a leak of a turbine stage.*

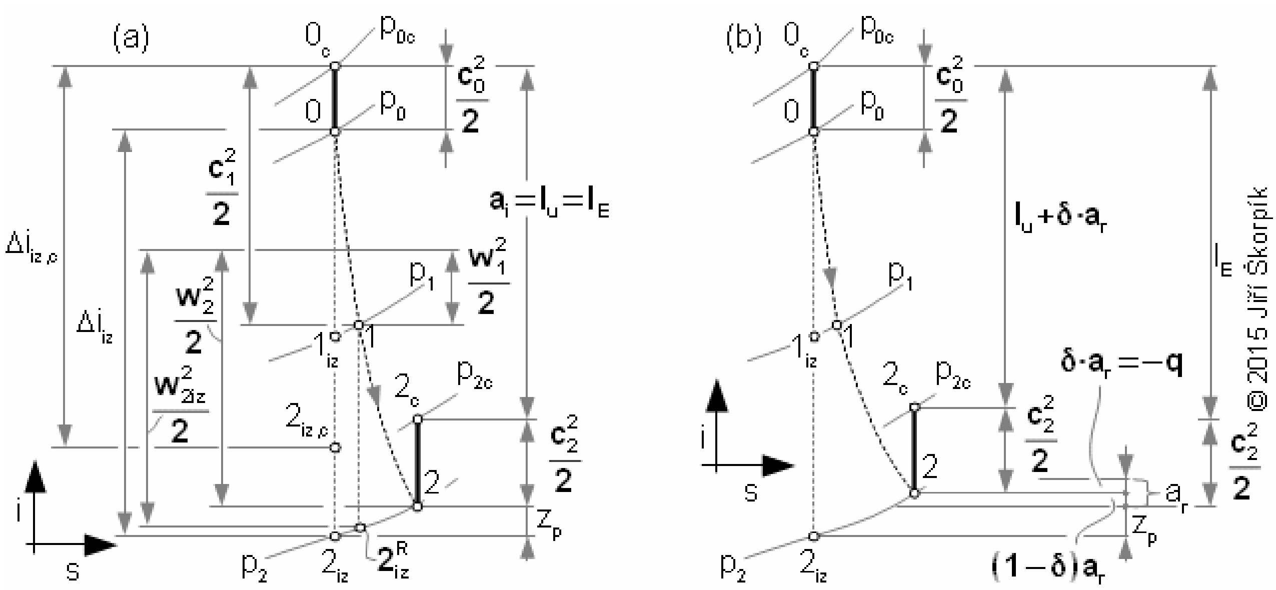
[3]*Other losses of the stage*

These losses are depend by type of a construction of the stage and its quality (inside one stage can be a few types of the other losses). More information is shown in the article 17. Losses in turbomachines.

In computing practice, the term spedific shaft l_u means the internal work of the stage without the influence of other losses (perfectly tight stage and without losses occurring at the root and tips of the blades, we can also speak of a stage with infinitely long blades). The internal specific work of stage a_i is then the actual specific work of stage including all losses ($l_u > a_i$).

i-s diagram of stage with taking into account rotor friction losses

For turbine stages be can constructed i-s diagram according the chapters 13. Adiabatic expansion inside heat turbine, 13. Polytropic expansion inside heat turbine:



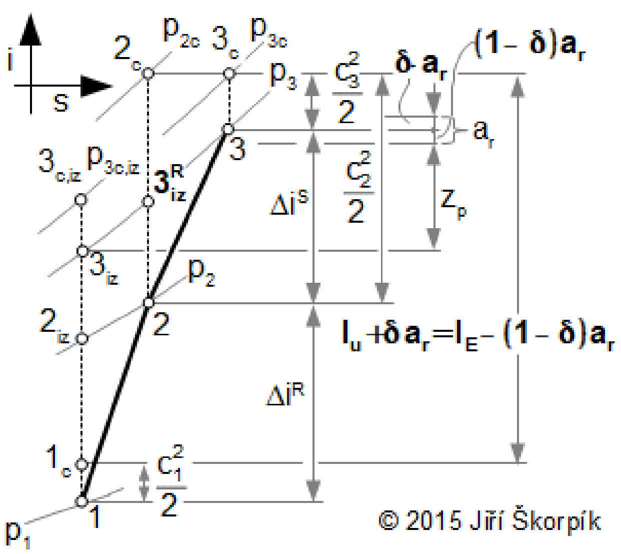
4.936 *i-s diagram of heat turbine stage.*

The figure shows expansion inside a reaction axial stage on a tested radius *r*. **(a)** case for $a_r<<l_E$ – negligible influence of rotor friction losses[(4)]; **(b)** case for $a_r>0$. **i** [J·kg^{-1}] specific enthalpy; **s** [J·kg^{-1}·K^{-1}] specific entropy; **p** [Pa] pressure; $\mathbf{z_p}$ [J·kg^{-1}] specific blade profile loss (friction between working fluid and surface of blades); **q** [J·kg^{-1}] specific heat of working gas transfer with surroundings. Subscript **iz** denotes the state of the working fluid at exit of the blade rows for case isentropic process, subscript denotes **c** stagnation state.

[(4)]*Remark*

For the indication of the kinetic energy of the relative velocities in the i-s diagram can to help equation for the specific shaft work. From this equation is evident, the sum of the specific shaft work l_E must be equal to the sum of the kinetic energy on the inlet of the stage and the difference of the kinetic energy of the relative velocities.

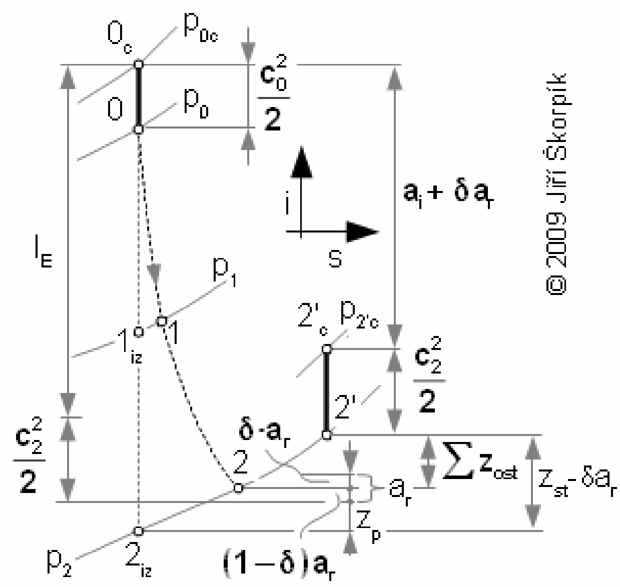
For stages of working machines be can constructed i-s diagram according the chapters 13. Adiabatic compression inside compressor, 13. Polytropic compression inside compressor:



5.719 *i-s diagram of compressor stage on radius r.*

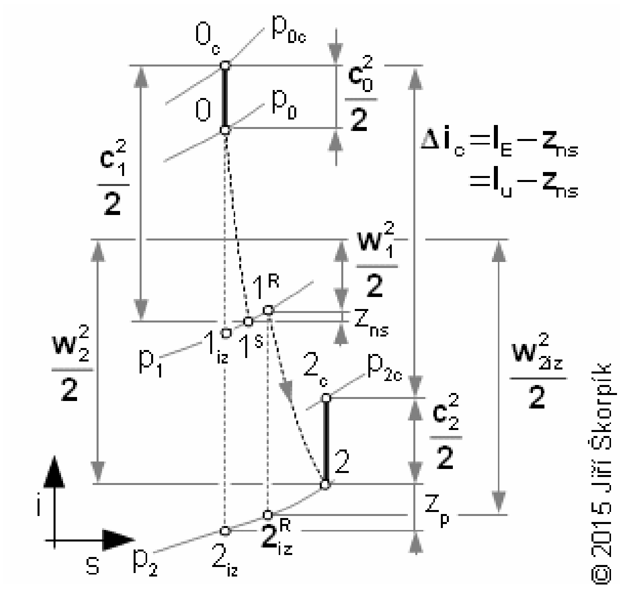
Total energy balance of stage

Total energy balance of the stage in i-s diagram shows all losses, enthalpy and the work of the working fluid at the exit for case ideal mixing:



6.319 *The specific internal work of a turbomachine stage on a tested radius r.* $\mathbf{z_{st}}$ [J·kg^{-1}] total losses of stage. The other losses of stage increases enthalpy from state *2* on the state *2′* at the exit.

If the other losses influences state of the working fluid inside core flow at exit first blade row then it can be influenced proces inside second blade row:



7.947 *Influence the other losses on the work shaft.* $\mathbf{z_{ns}}$ [J·kg^{-1}] specific loss through leaks of stator. This is case from *Figure 5* ($a_r \approx 0$ J·kg^{-1}). The flow inside core of stage is influenced by leaks on the stator blade row in this case. The working fluid from seal of the stator increases enthalpy at the inlet rotor blade row.

The shape of i-s diagram of flow through blade row of the stage corresponds to construction of stage. The i-s diagrams axial and diagonal stages are shown in article 19. Design of axials turbomachine stages, and in article 20. Design of radials turbomachine stages are shown i-s diagrams of radial stages.

For cases turbomachines without casing the flow mass of the working fluid beside the stream-tube of rotor are not included to the other losses z_{ost}.

Efficiency of stage

Two efficiencies of the stage are defined because two work of the stage are defined:

$$\eta_E=\frac{l_E}{e_0};\quad e_0=\underbrace{\kappa_0\frac{c_0^2}{2}}_{(a)}+\Delta i_{iz}-\underbrace{\kappa_2\frac{c_{2,iz}^2}{2}}_{(b)};\quad \eta_i=\frac{a_i}{e_0}$$

8.876 *The specific shaft work efficiency and the thermodynamic efficiency of a turbine stage.*
$\boldsymbol{\eta_E}$ [-] specific shaft work efficiency of turbine stage without other losses; $\mathbf{e_0}$ [J·kg^{-1}] specific energy of working fluid at inlet of stage; **(a)** [J·kg^{-1}] portion of specific kinetic energy of inlet velocity of working fluid, which is used inside stage[(5)]; **(b)** [J·kg^{-1}] portion of specific kinetic energy of exit velocity of working fluid, which is used inside next stage[(6)]; $\boldsymbol{\eta_{st}}$ [-] **internal (thermodynamic) efficiency** of stage. Approximate is true $c_{2,iz}=c_2$.

[(5)]*Remark*
The value of the coefficient κ_0 is in interval from 0 to 1. Usually it is required $\kappa_0=1$. The requirement $\kappa_0<1$ there are if the losses between a measuring point of the velocity c_0 (e.g. the exit previous stage) and a beginning of the blade row are not a portion of the losses of the stage. So, the value of the coefficient κ_0 is connected with definition of the boundary stage. For example, the boundary of the wind turbine stage a designer can define tightly in front of the rotor then vortex losses, which are developed between the inlet to the stream-tube and the rotor are not included in energy balance of the rotor and the value of the coefficient is $\kappa_0<1$.

[(6)]*Remark*
The value of the coefficient κ_2 is in interval from 0 to 1. For cases multi-stage turbomachines is $\kappa_2=1$ (in these cases the kinetic energy of the working fluid at the exit of the stage is not considered as a loss), only for case the last of the stages or one-stage machines is used $\kappa_2=0$.

For cases stages of working machines is usually defined the effective efficiency respectively isentropic efficiency of the stage. These efficiency are connected with static state of the working fliud at isentropic processes:

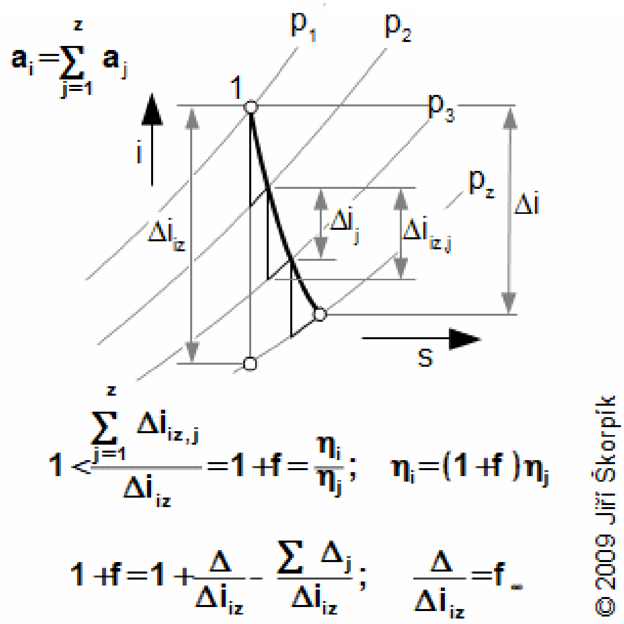
$$\eta_E=\frac{\Delta i_{iz}}{l_E}=\eta_{ef};\quad \eta_{iz}=\frac{\Delta i_{iz}}{a_i};\quad \eta_{iz,c}=\frac{\Delta i_{iz,c}}{a_i}$$

9.356 *The effective efficiency and isentropic efficiency of a working machine stage.*
$\boldsymbol{\eta_{ef}}$ [-] effective efficiency of working machine stage (usually specific shaft work is without other losses in this case); $\boldsymbol{\eta_{iz}}$ [-] isentropic efficiency of stage; $\boldsymbol{\eta_{iz,c}}$ [-] isentropic efficiency of stage in relation to stagnation state on exit of stage (for usually case $c_2=c_{2,iz}$ is true $\eta_{iz,c}=\eta_{iz}$).

The difference of the enthalpy of steam inside one stage of a steam turbine for isentropic process is *21,3 kJ·kg^{-1}*. The exit of the stage is the same as the velocity of steam at the inlet to the stage. The calculated blade profile losses of the stage are *3,3970 kJ·kg^{-1}* (the rotor blade row is geometrically same as the stator blade row). The calculated internal work of stage is *16,0744 kJ·kg^{-1}*. Calculate work shaft efficiency and the internal efficiency of this stage. This stage is the first stage of a multi-stage steam turbine. The solution of this problem is shown in the *Appendix 923*.
Problem 1.923

Efficiency of group of stages

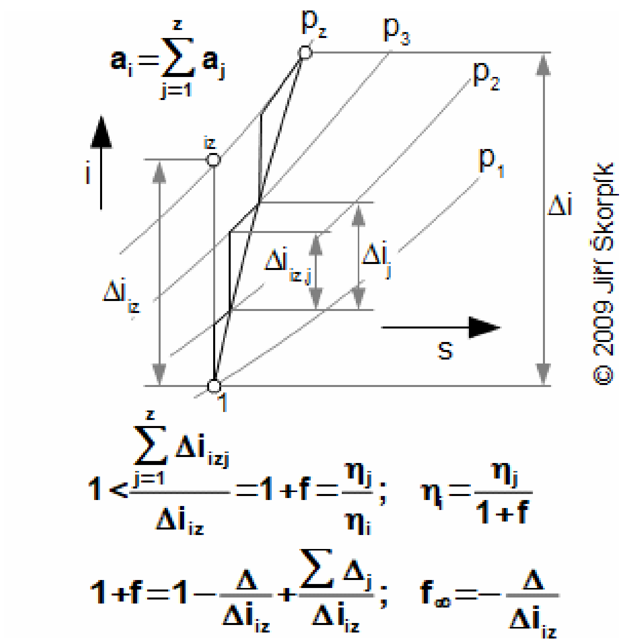
The sum of the energy balance all stages and the energy balance of other section of the turbine must be equal the energy balance of the stage section of the turbine as one entire:



10.116 *A multistage adiabatic expansion inside turbine.*
$\boldsymbol{\eta_j}$ [-] mean internal efficiency of individual stages; **1+f** [-] **reheat factor** (coefficient of the re-usable heat, *1,02* to *1,04* according [3]); **Δ** [J·kg^{-1}] re-usable heat of turbine; **1+f**$_\infty$ [-] reheat factor for case heat turbine with infinite number of stages; **z** [-] numbers of stages. The efficiency of stage section of multi-stage turbines η_i e.g. multi-stage steam turbines is higher than η_j of individual stages. These equation are derived for the same the enthalpy difference for all stages and for adiabatic expansion. For better illustration is not drawn the absolute velocity *c*. The derivation of this equation is shown in the *Appendix 116*.

It is evident, a portion of the heat from the loss processes inside previous stage is used inside next stage during expansion. Only the heat from the loss processes inside last stage of the heat turbine are not used. The re-usable heat influences as well as flow through one stage, because a portion of stator losses be can use in the rotor etc.

In case multi-stage turbocompressors, the resulting compression is composed of several sub-compression, their number is equal number of the stages inside turbocompressor:



11.121 *An multistage compression inside turbocompressor.*
1+f [-] **preheat factor** (coefficient of additional losses); $\boldsymbol{\Delta_j}$ [-] additional losses of one stage; **1+f**$_\infty$ [-] preheat factor in theoretical case turbocompressor with infinite number of stages. The efficiency η_i of multi-stage turbocompressor is less than η_j. These equation are derived for the same the enthalpy difference for all stages and for adibatic compresion. For better illustration is not drawn the absolute velocity *c*. The derivation of these equations is shown in the *Appendix 121*.

It is evident, the internal losses of one turbocompressor stage reduces the internal efficiency of next stage. The additional losses influences as well as flow through one stage, because losses of rotor increases losses of stator etc.

The air with temperature *15 °C* and pressure *0,1013 MPa* enters to a turbocompressor, on exit of the turbocompressor the air has temperature *293 °C* and pressure *0,802 MPa*. Calculate work a_i, a_{iz}, internal efficiency η_i and factor *1+f*. Number of stages of the turbocompressor is *12*. Use simplification c_p*=const.*. The solution of this problem is shown in the *Appendix 122*. This problem is published in [4].

Problem 2.122

References

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