APPENDIX 862: Fluid dynamics, Flow of gases and steam through nozzles, Calculation of position of shock wave in nozzle

This case is illustrated in FIGURE 105 (p. 12), which shows the ideal expansion with a shock wave in the nozzle, this is the case when the backpressure is p_{ec}. That is, supersonic expansion proceeds before the shock wave and subsonic compression proceeds after the shock wave. Estimation of the location of the shock wave origin can therefore be done by calculating the state quantities of the gas in the nozzle between the throat and the mouth (for selected locations) during expansion at the design condition. We will denote these states by 1x, as these will be the states before the shock wave. In the second step, we use the Rankine-Hugoniot equations (see article Mach number and high velocity flow effects) to calculate the states behind a possible wave if it were to occur at the location under investigation. We denote these states by the index 2x.

$$M_2^2=\frac{\frac{\kappa-1}{2}M_1^2+1}{\kappa\cdot M_1^2-\frac{\kappa-1}{2}};\quad \frac{T_2}{T_1}=\frac{1+\frac{\kappa-1}{2}M_1^2}{1+\frac{\kappa-1}{2}M_2^2};\ \frac{p_2}{p_1}=\frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}.$$

Entered parameter of the problem is:

Δp
0,55

p [MPa].

Other known parameters obtained from the solution of PROBLEM 104, (p. 10) and PROBLEM 102, (p. 6) are given in the text.

A862/2: The pressure p_{1x} can range from the critical pressure in the nozzle to the pressure behind the nozzle at the design state p_{en}. The critical pressures can be found from the critical pressure ratio ε^*_s and the total pressure p_{is} – both parameters are given in PROBLEM 102, (p. 6). Next, the pressure ratio ε_{sx} and from there the velocities V_{1x} can be calculated using EQUATION 101, (p. 4), and the working gas parameters can be read off from PROBLEM 102, (p. 6).

Next, the specific volume v_{1x} can be calculated using the isentropy equation in A102/6:

$$p_{is}\cdot v_{is}^{\kappa}=p_{1x}\cdot v_{1x}^{\kappa}.$$

The temperature t_{1x} can be calculated from the pressure p_{1x} and the specific volume v_{1x} using the ideal gas equation of state (A102/6):

$$p_{1x}\cdot v_{1x}=r\cdot T_{1x}.$$

The speed of sound a_{1x} can be calculated using the equation (A104/3):

$$a_{1x}=\sqrt{\kappa \cdot r \cdot T_{1x}}.$$

The actual flow cross section A_x can be determined from the continuity equation (EQUATION 334, p. 4). The x-coordinate can also be calculated from the flow area A_x using EQUATIONS 703, (p. 10) and the nozzle parameters in PROBLEM 104, (p. 10), assuming the nozzle is circular in cross section and the individual radii can be determined from the equation for the area of a circle:

$$r_{1x}=\sqrt{\frac{A_{1x}}{\pi}}.$$

Nozzle parameters from Problem 104, (p. 10):

p_{is}	T_{is}	v_{is}	p_{en}	$\dot{m}$	ε^*_s	r	κ	t
1,1848	654,09	158,44	0,25	2,8086	0,5283	287	1,4	0,0727
α	r_t							
10	2,1883							

p [MPa]; T [K]; v [dm^3·kg^{-1}]; m [kg·s^{-1}]; ε [1]; r [J·kg^{-1}·K^{-1}]; κ [1]; t, r_t [cm]; α [°]

	1	2	3	4	5	6	7	8
p_{1x}	625,95	550	500	450	400	350	300	250
ε_{s1x}	0,5283	0,4642	0,4220	0,3798	0,3376	0,2954	0,2532	0,2110
V_{1x}	467,97	508,66	535,80	563,51	592,04	621,76	653,11	686,73
v_{1x}	249,92	274,11	293,42	316,36	344,13	378,56	422,63	481,41
t_{1x}	271,93	252,15	238,04	222,88	206,47	188,51	168,62	146,20
a_{1x}	467,99	459,42	453,21	446,44	438,99	430,69	421,31	410,48
M_{1x}	1,0000	1,1072	1,1822	1,2622	1,3487	1,4436	1,5502	1,6730
A_{1x}	14,999	15,135	15,381	15,768	16,325	17,100	18,175	19,689
r_{1x}	2,1850	2,1949	2,2127	2,2403	2,2796	2,3331	2,4052	2,5034
x	0,0000	0,1489	0,3516	0,6677	1,1162	1,7279	2,5527	3,6751

p [kPa]; ε [1]; V, a [m·s^{-1}]; v [dm^3·kg^{-1}]; t [°C]; M [1]; A [cm^2]; r, x [cm]

A862/3: Now we have the necessary parameters at the input to the shock wave if it were to occur at the x-coordinate. The output of such the wave must have the parameters determined by the Rankine-Hugoniot equations in A862/1:

	1	2	3	4	5	6	7	8
M_{2x}	1,0000	0,9062	0,8534	0,8059	0,7624	0,7220	0,6840	0,6477
t_{2x}	271,92	288,66	297,77	305,75	312,83	319,18	324,97	330,31
p_{2x}	625,91	694,91	731,99	761,44	782,14	792,65	791,06	774,67
v_{2x}	249,93	232,03	223,85	218,2	215,02	214,47	217	223,57
V_{2x}	468	430,57	408,75	388,66	369,92	352,24	335,34	318,92

M [1]; t [°C]; p [kPa]; v [dm^3·kg^{-1}]; V [m·s^{-1}]

A862/4: Now the isentropic compression parameters for cases 1 to 8 can be calculated, which should follow from the shock wave to the nozzle outlet. From the available data and the new backpressure p_e, we can calculate what the outlet flow area of the nozzle A_{eR} would have to be for the flow to be in equilibrium.

The case where A_{eR} is closest to the actual nozzle flow area A_e will also be closest to reality see below.

The actual nozzle cross section from Problem 104, (p. 10) is:

A_e
19,689

[cm²]

We calculate the new backpressure p_e from the given pressure change Δp and the specific volume v_{eR} from the isentropy equation:

$$p_{2x} \cdot v_{2x}^{\kappa} = p_e \cdot v_{eR}^{\kappa}.$$

Next, the velocity V_{eR} can be determined from EQUATION 104, (p. 4) and then the flow cross section A_{eR} from EQUATION 334, (p. 4):

	1	2	3	4	5	6	7	8
p_e				800,00				
v_{eR}	209,75	209,83	210,08	210,63	211,58	213,06	215,26	218,49
ε_R	1,28	1,15	1,09	1,05	1,02	1,01	1,01	1,03
V_{eR}	373,48	372,89	370,94	366,78	359,48	347,76	329,53	300,86
A_{eR}	15,77	15,8	15,91	16,13	16,53	17,21	18,35	20,4

p [kPa]; v [dm³·kg⁻¹]; ε [1]; V [m·s⁻¹]; A [cm²]

A862/5: From the last table, it is clear that the shock wave will occur somewhere in the location of nozzles marked 7 and 8, i.e. in the section between 2,5527 and 3,6751 cm. A more precise location can be obtained by dividing the section 7-8 into several parts.