APPENDIX 515: Fluid dynamics, Flow of gases and steam through nozzles, Derivation of equation for maximum mass flow of gas through nozzle

The mass flow of gas through the nozzle m can be determined according to EQUATION 334, (p. 4). The maximum possible flow through the nozzle occurs at a certain pressure ratio, denoted as ε^*_s. This pressure ratio is obtained by deriving EQUATION 334, (p. 4) with respect to the pressure ratio ε_s, and this derivative must equal 0:

$$\frac{\mathrm{d}\dot{m}}{\mathrm{d}\varepsilon_s}=0=\frac{1}{2}A_e\sqrt{\frac{2\cdot\kappa}{\kappa-1}}\sqrt{\frac{p_{is}}{v_{is}}}\left(\varepsilon_s^{\frac{2}{\kappa}}-\varepsilon_s^{\frac{\kappa+1}{\kappa}}\right)^{-\frac{1}{2}}\cdot\left(\frac{2}{\kappa}\varepsilon_s^{\frac{2-\kappa}{\kappa}}-\frac{\kappa+1}{\kappa}\varepsilon_s^{\frac{1}{\kappa}}\right)=0.$$

A515/2: The last equation will be equal to zero if this part of the equation is equal to zero:

$$\frac{2}{\kappa}\varepsilon_s^{\frac{2-\kappa}{\kappa}}-\frac{\kappa+1}{\kappa}\varepsilon_s^{\frac{1}{\kappa}}=0.$$

The ratio of the total pressures ε_s in the last equation is the ratio at which the maximum mass flow through the nozzle occurs:

$$\varepsilon^{\frac{2-\kappa}{\kappa}-\frac{1}{\kappa}}=\frac{\kappa+1}{\kappa}\frac{\kappa}{2}$$

$$\varepsilon_s^{\frac{1-\kappa}{\kappa}}=\frac{\kappa+1}{2}$$

$$\varepsilon_s=\left(\frac{\kappa+1}{2}\right)^{\frac{\kappa}{1-\kappa}}=\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}.$$

Hence the pressure ratio you are looking for:

$$\varepsilon^*_s=\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}=\frac{p^*}{p_{is}}.$$