APPENDIX 334: Fluid dynamics, Flow of gases and steam through nozzles, Derivation of equation for mass flow of gas through nozzle

The mass flow m can be derived from the continuity equation. This equation can be applied to the inlet or outlet of the nozzle. In practice, the second case is preferable because the nozzle exit is more clearly defined than the nozzle entrance, so we write the continuity equation in the form:

$$\dot{m} = A_e \cdot V_e \frac{1}{v_e}.$$

A334/2: Next, we can proceed by substituting the velocity at the nozzle exit V_e from EQUATION 101, (p. 4):

$$\dot{m} = A_e \sqrt{\frac{2\kappa}{\kappa - 1} r \cdot T_{is} \left[1 - \left(\frac{p_e}{p_{is}} \right)^{\frac{\kappa - 1}{\kappa}} \right]} \frac{1}{v_e}.$$

The product of the individual gas constant r and the total temperature T_{is} can also be written according to the equation of state [Škorpík, 2024] as follows:

$$r \cdot T_{is} = p_{is} \cdot v_{is}.$$

Hence for the mass flow of gas through the nozzle:

$$\dot{m} = A_e \sqrt{\frac{2\kappa}{\kappa - 1} p_{is} \cdot v_{is} \left[1 - \left(\frac{p_e}{p_{is}} \right)^{\frac{\kappa - 1}{\kappa}} \right]} \frac{1}{v_e}.$$

A334/3: The specific gas volume at the nozzle exit can be determined from the isentropy equation [Škorpík, 2024]:

$$\frac{1}{v_e} = \left(\frac{p_e}{p_{is}} \right)^{\frac{1}{\kappa}} \frac{1}{v_{is}}.$$

A334/4: Combining the above equations we obtain:

$$\dot{m} = A_e \sqrt{\frac{2\kappa}{\kappa - 1} p_{is} \cdot v_{is} \left[1 - \left(\frac{p_e}{p_{is}} \right)^{\frac{\kappa - 1}{\kappa}} \right]} \left(\frac{p_e}{p_{is}} \right)^{\frac{1}{\kappa}} \frac{1}{v_{is}}$$

$$\dot{m} = A_e \sqrt{\frac{2\kappa}{\kappa - 1}} \sqrt{\frac{p_{is}}{v_{is}}} \sqrt{\left(\frac{p_e}{p_{is}} \right)^{\frac{2}{\kappa}} - \left(\frac{p_e}{p_{is}} \right)^{\frac{\kappa + 1}{\kappa}}} = A_e \sqrt{\frac{p_{is}}{v_{is}}} \chi_m$$

$$\chi_m = \sqrt{\frac{2\kappa}{\kappa - 1}} \sqrt{\left(\frac{p_e}{p_{is}} \right)^{\frac{2}{\kappa}} - \left(\frac{p_e}{p_{is}} \right)^{\frac{\kappa + 1}{\kappa}}}.$$