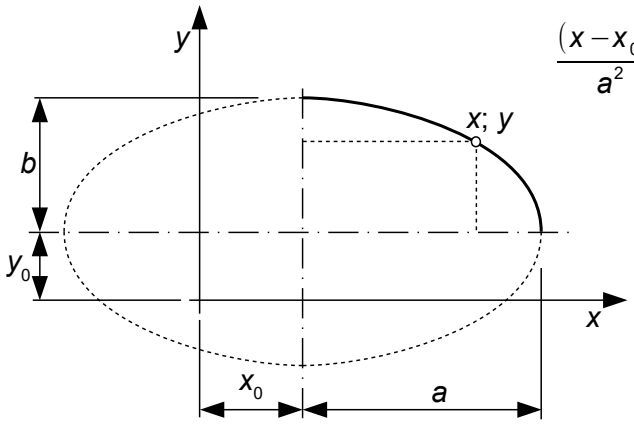
# **APPENDIX 162:** Fluid dynamics, Flow of gases and steam through nozzles, Derivation of Bendemann ellipse equation

*The derivation is based on the general ellipse equation. In the attached figure of the ellipse, a quarter arc is marked, which should be a substitute for the m-$\varepsilon_s$ chart:*




$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

**A162/2:** The values of $x$; $y$ can be assumed to be the similarities with FIGURE 515, (p. 5) as follows:

$$x \approx \dot{m};\ y \approx \varepsilon_s;\ x_0=0;\ y_0=\varepsilon_s^*;\ a=\dot{m}^*;\ b \approx 1-\varepsilon_s^*.$$

**A162/3:** Hence the Bendemann ellipse equation:

$$\frac{\dot{m}^2}{\dot{m}^{*2}}+\frac{(\varepsilon_s-\varepsilon_s^*)^2}{(1-\varepsilon_s^*)^2}=1.$$

The mass flow through the nozzle can therefore be approximately calculated by separating the mass flow from the Bendemann ellipse equation:

$$\dot{m} \approx \dot{m}^* \sqrt{1-\frac{(\varepsilon_s-\varepsilon_s^*)^2}{(1-\varepsilon_s^*)^2}},$$

$$\varepsilon_s=\frac{p_e}{p_{is}};\ \varepsilon_s^*=\frac{p^*}{p_{is}},$$

$$\dot{m} \approx \dot{m}^* \sqrt{1-\frac{(p_e-p^*)^2}{(p_{is}-p^*)^2}}.$$